CUSUM TEST : A PREDICTIVE TEST OF STRUCTURAL CHANGE

RECURSIVE RESIDUALS

CONSIDER THE MODEL

$$y_t = \sum_{j=1}^{k} x_{jt} \beta_j + u_t$$

$$y_t = x_t \beta + u_t \qquad t = 1, 2, \ldots, n$$

$$x_t = \begin{pmatrix} x_{1t} \\ x_{2t} \\ x_{kt} \end{pmatrix}' = (x_{1t} \ x_{2t} \ldots, x_{kt})$$

$$\beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_k \end{pmatrix}.$$

AT ANY GIVEN t, DEFINE

$$X_{t-1} = \begin{bmatrix} x_{11} \ldots & x_{k1} \\ x_{12} & x_{k2} \\ x_{1t-1} & x_{kt-1} \end{bmatrix}$$

$$y_{(t-1)} = \begin{bmatrix} y_1 \\ y_2 \\ y_{t-1} \end{bmatrix}$$

GET THE OLS ESTIMATOR

$$\hat{\beta}_{(t-1)} = (X_{t-1}' X_{t-1})^{-1} X_{t-1}' y_{(t-1)}.$$

NOW USE THIS $\hat{\beta}_{(t-1)}$ ALONG WITH THE ACTUAL $x_t = (x_{1t} \ x_{2t} \ x_{kt})$ TO GET A FORECAST FOR y_t

$$y_{ft} = x_t \hat{\beta}_{(t-1)}$$

THE FORECAST YIELDS THE PREDICTION ERROR

$$e_t = y_t - x_t(\beta_{(t-1)}) .$$

NOTE THAT $E(e_t) = 0$ (AS SHOWN EARLIER)

$$Var(e_t) = \sigma^2\left(1 + x_t'\left(X_{t-1}'X_{t-1}\right)^{-1}x_t'\right)$$

DEFINE $$\omega_t = \frac{e_t}{\sqrt{var(e_t)}}$$

UNDER H_0 THAT THE COEFFICIENTS REMAIN STABLE OUT OF SAMPLE

$$\omega_t \sim N(0, \sigma^2)$$

THUS $$\frac{\omega_t}{\sigma} \sim N(0,1)$$

CUSUM <u>TEST</u>:

CONSIDER THE CUMULATIVE SUM OF RESIDUALS

$$W_t = \sum_{r=K+1}^{t} \frac{\omega_r}{\hat{\sigma}}$$

HER $$\hat{\sigma}^2 = \frac{1}{n-(K+1)} \sum^{n} (\omega_r - \bar{\omega})^2$$

$$\bar{w} = \frac{1}{n-K} \sum_{r=K+1}^{n} \omega_r$$

NOTE $$\frac{\omega_r}{\sigma} \sim N(0,1)$$

THUS W_t HAS AN APPROXIMATE NORMAL DISTRIBUTION WITH MEAN 0 & VARIANCE $(t-(K+1))$.

BROWN, DURBIN, & EVANS DEVELOPED CONFIDENCE BOUNDS OF THE CUMULATIVE SUM BY PLOTTING 2 LINES THAT CONNECT THE POINTS $(k, \pm a(n-k)^{1/2})$ & $(n \pm 3a(n-k)^{1/2})$.

FOR THE 95% LEVEL OF SIGNIFICANCE,

$\alpha = 0.948$.

THE CUMULATIVE SUM ~~IS~~ W_t IS PLOTTED AGAINS t FOR $t > k$.

IF THE LINE STRAYS OUTSIDE THE BOUNDS, MODEL INSTABILITY IS IMPLIED.