

Answer the questions fully to your best ability. Use the space provided. If you run out of room, use the backsides. No partial credit will be given if you do not show the steps of your calculations! **Write as neatly as possible!**

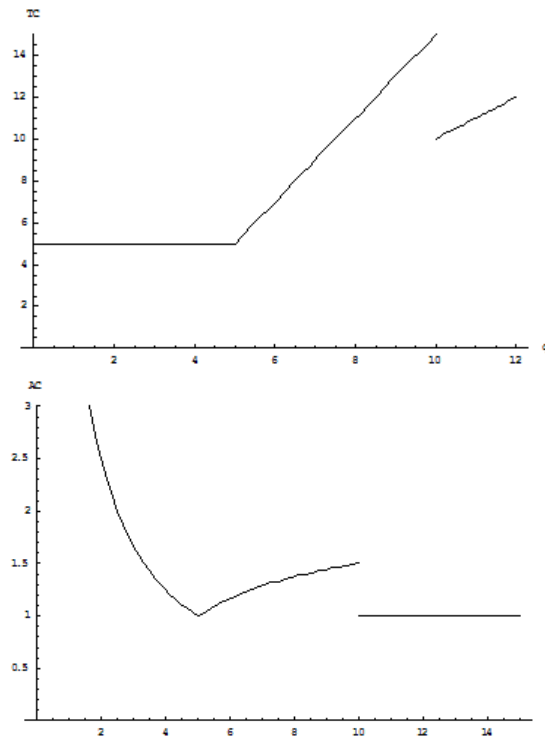
Name: _____

- (10) 1. Let's say that the total cost of the firm (TC) depends on the amount of goods produced (q) in this particular way:

$$\begin{cases} TC = 5, & \text{if } 0 \leq q \leq 5 \\ TC = -5 + 2q, & \text{if } 5 < q < 10 \\ TC = q, & \text{if } q \geq 10 \end{cases}$$

Draw the total costs, average costs, and marginal costs curves.

Solution:



Marginal costs are zero in the first segment, 2 in the second, and 1 in the third.

- (5) 2. Characterize this function: $y(x) = ax^2 + bx + c$. Is it continuous? Is it increasing or decreasing? Is it concave or convex?

Solution: The answer depends on the specific values of the parameters $\{a, b, c\}$. If $a \neq 0$, it is a parabola. If $a > 0$, it is convex, if $a < 0$, it is concave. If $a = 0$, it collapses to a linear equation. In that case, if $b > 0$ it is increasing, if $b < 0$ it is decreasing. Note, that if $a = b = 0$, the function is constant at c .

- (5) 3. Evaluate the following expression: $\lim_{x \rightarrow 1} \frac{1-2x+x^2}{1-x} =$

Solution: Note that $\frac{1-2x+x^2}{1-x} = \frac{(1-x)(1-x)}{1-x} = 1 - x$. Thus $\lim_{x \rightarrow 1} \frac{1-2x+x^2}{1-x} = 0$

4. Solve the following equations:

(10) (a) Solve for x : $10x^2 + 50x + 70 = 0$

Solution: In this question we are looking for the “roots” of the quadratic function. In other words we want to find the points where the function $y = 10x^2 + 50x + 70$ intersects the horizontal axis. The solution for quadratic equation is: $x_{1,2} = \frac{-50 \pm \sqrt{2500 - 2800}}{20}$. Note, that we can not take a root of a negative number. This means that the parabola defined by $y = 10x^2 + 50x + 70$ never intersects the horizontal axis.

(10) (b) Solve for x , y and z :

$$\begin{cases} x = 6z - 4a + 2 \\ y = 4z + 1 \\ x = y^2 \end{cases}$$

Solution: There are many ways you can solve this system of equations. For example: since $x = y^2$, it must be the case that $6z - 4a + 2 = (4z + 1)^2$, or $16z^2 + 2z + [4a - 1] = 0$. A solution to this quadratic equation results in: $z_{1,2}^* = \frac{-1 \pm \sqrt{17 - 64a}}{16}$. Solution for x and y can be obtained by simple substitution of z^* into the first or second equation.

5. Simplify the following expressions:

(3) (a) $x^2y^3 \cdot x^3 \div xy^2 \cdot x^{-2}y$

Solution: x^2y^6

(3) (b) $\{(x + 1)^4(2x - 1) + (x + 1)^4(2 - x)\}^5$

Solution: $(x + 1)^{25}$

(4) 6. Use the rules of logarithms to simplify the following: $\ln\left(\frac{1}{e^5}[x^\alpha y^{-\beta}]^2\right)$

Solution: $-5 + 2\alpha \ln(x) - 2\beta \ln(y)$

- (10) 7. Bank Highrates is still offering the deposit-incentive annual interest rate of 8%. Its upstart competitor, Bank Betterrates, is offering a similar incentive program but will pay an effective rate of 8.25% on an equivalent deposit. Which bank is offering its investors a better deal?

Solution: The relationship between the annual interest rate r_A and effective interest rate r_E is: $r_E = e^{r_A} - 1$. It appears that bank Highrates is offering a better deal.

8. Economist Keynes proposed one particular way to look at the economy: The demand for good in the economy is comprised of consumption demand (C), investment demand (I) and government consumption (G). This way aggregate demand is defined as $AD = C + I + G$. Keynes assumes that consumers consume a fixed fraction of their after-tax income, so that consumption is: $C = a + b(Y - T)$. Similarly, investment is also a fixed fraction of gross income: $I = d + sY$. Economy is in equilibrium, whenever amount of goods produced is equal to the amount of goods demanded. Since there is no money in this model, amount of goods produces is also the income (Y). So that in equilibrium $Y = AD$.

- (5) (a) Find an equilibrium output (Y^*) in this economy.

Solution: $Y^* = \frac{a+d-bT+G}{1-b-s}$

- (5) (b) How much does the Y^* change if government increases consumption (G) by 1?

Solution: $\frac{1}{1-b-s}$

- (5) (c) How much does the Y^* change if taxes (T) go up by 1?

Solution: $\frac{-b}{1-b-s}$

- (5) (d) Write this problem in matrix form. The endogenous variables are: Y and AD .

$$\text{Solution: } \begin{pmatrix} -b-s & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} Y \\ AD \end{pmatrix} = \begin{pmatrix} a - bT + d + G \\ 0 \end{pmatrix}$$

9. Consider the matrices:

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \\ 3 & -1 \end{pmatrix}, B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- (3) (a) Is $A + I$ defined? If so, calculate $A + I$. If undefined, explain why.

Solution: Undefined. Matrices are not conformable.

- (3) (b) Is $A \cdot B$ defined? If so, calculate $A \cdot B$. If undefined, explain why.

$$\text{Solution: } \begin{pmatrix} a + 2c & b + 2d \\ 4a + 3c & 4b + 3d \\ 3a - c & 3b - d \end{pmatrix}$$

- (3) (c) Is $B \cdot A$ defined? If so, calculate $B \cdot A$. If undefined, explain why.

Solution: Undefined. Matrices are not conformable.

- (3) (d) Is $A \cdot I$ defined? If so, calculate $A \cdot I$. If undefined, explain why.

Solution: Undefined. Matrices are not conformable.

- (3) (e) Is $A \cdot A'$ defined? If so, calculate $A \cdot A'$. If undefined, explain why.

$$\text{Solution: } \begin{pmatrix} 5 & 10 & 1 \\ 10 & 25 & 9 \\ 1 & 9 & 10 \end{pmatrix}$$

- (3) (f) Is $A'B - B$ defined? If so, calculate $A'B - B$. If undefined, explain why.

Solution: Undefined. Matrices are not conformable.

