Answer the questions fully to your best ability. Use the space provided. If you run out of room, use the backsides. No partial credit will be given if you do not show the steps of your calculations! **Write as neatly as possible!**

Name: ______________________________

1. Let's say we want to model household behavior using a utility function of this particular form:

$$U(c) = A\ln(c)\cdot\sqrt{c}$$

where c stands for consumption, and U is utility level.

(5) (a) Does this functional form have a property that utility is increasing in consumption?

Solution: A function is increasing if it's first derivative is positive, thus we need to take a derivative of $U(c)$ and sign it:

$$\frac{dU(c)}{dc} = \frac{A}{c}\cdot\sqrt{c} + \frac{1}{2\sqrt{c}}\cdot A\ln(c) = \frac{A(2+\ln(c))}{2\sqrt{c}} > 0, \text{ if } A > 0$$

So that $U(c)$ is increasing in c.

(5) (b) Does this functional form have a property of diminishing marginal utility?

Solution: Marginal utility (MU) is simply the derivative of the utility function, which we obtained in part (a). If we want to show that marginal utility is diminishing, we have to show that the derivative of MU with respect to c is negative:

$$\frac{d^2U}{dc^2} = \frac{A/c\cdot 2\sqrt{c} - 1/\sqrt{c}\cdot[A(2+\ln(c))]}{4c} = -\frac{A\ln(c)}{4c^{3/2}} < 0, \text{ since } c \geq 0$$

(10) 2. Let's say we want to find a minimum of this disutility function:

$$D(x, y) = 2x^2 + 2y^2 - 2xy - 5(x + y)$$

(5) (a) The first order conditions for a minimum are: $D_x = 0$ **and** $D_y = 0$. Find these first order conditions.

Solution:

$$\begin{cases} D_x = 4x - 2y - 5 = 0 \\ D_y = 4y - 2x - 5 = 0 \end{cases}$$

(5) (b) Since we have to solve a system of equations, I propose to write it in matrix form, like $Av = w$, where A is a 2x2 matrix of parameters, v is a 2x1 matrix of variables, and w is 2x1 matrix of the constants. Write the system of equations that you got in (a) in matrix form.

Solution:

$$\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

(5) (c) Solve this system of equations using Cramer's rule.

Solution:

$$x = \frac{\begin{vmatrix} 5 & -2 \\ 5 & 4 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ -2 & 4 \end{vmatrix}} = \frac{5}{2}, \text{ and } y = \frac{\begin{vmatrix} 4 & 5 \\ -2 & 5 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ -2 & 4 \end{vmatrix}} = \frac{5}{2}$$

(5) (d) Because most probably we made some kind of a sloppy mistake, lets solve the problem using another method and see if answers match. Since the problem that we are solving is $Av = w$ (in matrix form), we could pre-multiply both sides of equality with an inverse of A: $A^{-1}Av = A^{-1}w$. Since $A^{-1}A$ gives us identity matrix, $v = A^{-1}w$. So we need to find an inverse of A, multiply it with w and we get the answer! Find an inverse of A, and carry out the multiplication $A^{-1}w$. You should get the same answer as in part (c).

Solution:

Determinant of A: $|A| = 12,$ Cofactor matrix: $C = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$, thus $C' = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

So that:

$$A^{-1} = \frac{1}{|A|} \cdot C' = \frac{1}{12} \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix}.$$

Solution:

$$v = A^{-1}w = \begin{bmatrix} 1/3 & 1/6 \\ 1/6 & 1/3 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \end{bmatrix} = \begin{bmatrix} 5/2 \\ 5/2 \end{bmatrix}.$$

Apparently we did not make any mistake. That is SO strange...

3. Marius got a job offer from Bestever University. While Marius would like to ask for as high a salary (s) as possible, he understands that if he asks for a very huge salary, the school most probably will tell him to look for a another job. The probability of this happening (p) depends on the salary asked: $p(s) = 1 - e^{-s}$. If this actually happens, Marius will have to take a job at Starbucks for m per year.

   Having thought out everything carefully Marius realizes that there is no way to make 100% sure he lands a job at Bestever University, (unless he asks for a 0 salary). Thus he decides to maximize his *expected* income (EI):

$$EI = [1 - p(s)] \cdot s + p(s) \cdot m$$

(10) (a) What salary Marius should ask? (Find the salary that would maximize expected income)

Solution: First order condition for a maximum is $\frac{dEI}{ds} = 0$:

$$\frac{dEI}{ds} = -p'(s) \cdot s + (1 - p(s)) + p'(s) \cdot m = 0$$

Since $p'(s) = e^{-s}$:

$$\begin{aligned} -e^{-s}s + 1 - (1 - e^{-s}) + me^{-s} &= 0, \text{ or} \\ -e^{-s}(s - 1 - m) &= 0 \\ s - 1 - m &= 0, \text{ since } -e^{-s} > 0 \\ s^* = m + 1 & \end{aligned}$$

(10) (b) How do we make sure that the answer in (a) is actually a maximum, and not a minimum? That would be an awful mistake!

Solution: The second order condition for a maximum is $\frac{d^2EI}{ds^2} < 0$:

$$\frac{d^2EI}{ds^2} = e^{-s}(s - 1 - m) - e^{-s} = e^{-s}(s - 2 - m)$$

How do we know that this expression is negative? Well, from part (a) we know

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that the optimum salary (s^*) is $s^* = m + 1$. So that:

$$\frac{d^2EI}{ds^2} = e^{-(m+1)}(m + 1 - 2 - m) = -e^{-(m+1)} < 0.$$

Thus, $s^* = m + 1$ is the salary that maximizes expected income.

Grade table

Question:	1	2	3	Total
Points:	10	30	20	60
Score:				