Some of these problems will appear on the midterm. If not exactly the same, but very similar. It is your responsibility to make sure you understand how to solve them. If you do not understand, it is my responsibility to explain. Thus do not hesitate to ask questions. **Don't forget to write as neatly as possible!**

Name: ____________________

1. Find the determinants of these matrixes:

   (a) $\begin{pmatrix} 5 & 0 & 4 \\ 0 & 3 & 1 \\ 2 & 6 & 0 \end{pmatrix}$

   (b) $\begin{pmatrix} 1/2 & -1 & 0 \\ 1 & -2 & 0 \\ 2 & -4 & 0 \end{pmatrix}$

   (c) $\begin{pmatrix} 1 & \lambda & -1 \\ 0 & \phi & 1 \\ -1 & 0 & 1 \end{pmatrix}$

   (d) $\begin{pmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ 0 & d_{22} & d_{23} & d_{24} \\ 0 & 0 & d_{33} & d_{34} \\ 0 & 0 & 0 & d_{44} \end{pmatrix}$

2. Find the inverse of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

3. Suppose that $y = f(x_1, x_2, x_3)$, where all of the partial derivatives f_1, f_2, and f_3 are strictly negative. However, $x_2 = g(x_1)$, where $g'(x_1) > 0$; and $x_3 = h(x_2)$, where $h'(x_2) > 0$. Show that the total derivative of y with respect to x_1 is strictly negative. Be sure to account for all of the ways a change in x_1 will lead to a change in y.

4. Two goods x and y give Jim utility. The function that est describes this relationship is:

$$U(x, y) = 1 - e^{-xy}.$$

   (a) For nonnegative (x, y) combinations, find the range of this utility function.

   (b) Find the marginal utility of each good and show that each expression is positive.

   (c) Show that the marginal utility of x is a decreasing function of x.

   (d) Show that Young's theorem holds for this function.

5. The total transportation cost (T) of shipping a product depends on the weight (w) of the product in tons, the number of miles (x) the good is shipped, and the transport rate (r) per ton-mile. In particular: $T(w, x, r)$. Where $T_w > 0$, $T_x > 0$, and $T_r > 0$. However, because of "volume discounts," the transportation rate varies with the weight and distance shipped, as given by the function: $r(w, x)$, where $r_w < 0$ and r_x¡0.

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(a) Given these relationships, find the expressions for the total derivatives: $\frac{dT}{dw}$ and $\frac{dT}{dx}$.

(b) What further restrictions on the partial derivatives of T(w,x,r) and r(w,x) would be needed to ensure that the total derivatives in (a) are strictly positive?

6. A paper company uses a single input (x) to produce units of computer paper (q), according to the following production function:

$$q = Ax^a \quad A > 0, 0 < a < 1.$$

Unfortunately, the production of this good also results in units of pollution (z). The amount of z is proportional to q, but can be reduced if the firm spends money on pollution control. The amount of pollution control expenditure is denoted by c. The relationship between various combinations of (q, c) is:

$$z = Bq - c^b \quad B > 0, 0 < b < 1.$$

The government has imposed a tax on pollution of $t > 0$ dollars per unit of z. Assuming that the firm faces a price of p dollars per unit of q and must pay w dollars per unit of x, the firm must choose some level of inputs (x) and some level of pollution control expenditure (c) that will maximize its profit. (Assume that $p > tB$.)

(a) Using the above information, construct the expression for the firm's profit (revenues minus input costs, pollution taxes, and pollution control costs) as a function of x and c and parameters of the model.

(b) Give the first-order conditions for a maximum, and solve for the reduced-form expressions for x^* and c^*.

(c) <SKIP THIS PART> Give the second-order conditions for a maximum, and verify that these conditions are satisfied in this problem.

(d) Use the reduced-form expressions (part b) to determine how an increase in the pollution tax (t) would affect the profit-maximizing levels of input (x^*) and pollution control expenditures (c^*).

(e) Find the expressions for z^*, the level of pollution associated with profit-maximizing behavior.

(f) Suppose that you are the government authority responsible for this pollution tax system, and that you want to maximize total pollution tax revenues (R) collected from this firm by setting t. Assuming that the authority understands the firm's behavior, construct this new maximization problem and explain what the authority must do. You need not solve for $t*$.

7. A local jazz group has recorded a new tune. The group decides to market the record to the public for the same price (p) per record. They also will allow radio shows to paly the record for a royalty fee (r) per play. Thus, they have two potential sources of revenue: record sales and royalties. They wish to maximize the combined revenue (R) from these two sources. The problem is made a little bit more interesting by the fact

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that the number of records sold (x) is a function of the price charged (p) and the number of times (z) the record is played on the radio:

$$x(p, z), \quad x_p < 0, x_z > 0.$$

Also, the number of times the record is played on the radio (z) is a function of the royalty fee the group charges:

$$z(r), \quad z_r < 0.$$

The group needs to select a price (p) and royalty fee (r) to maximize:

$$R(p, r) = p \cdot x[p, z(r)] + r \cdot z(r)$$

(a) Derive the first-order conditions for maximizing $R(p, r)$.

(b) Now, suppose that the following linear functional forms are introduced:

$$\begin{aligned} x(p, z) &= a - bp + cz, \quad & a, b, c > 0 \\ z(r) &= d - hr, & d, h > 0 \end{aligned}$$

Using these functions and your earlier first-order conditions, find the reduced-form expressions for p^* and r^*, as functions of the parameters (a,b,c,d,h).

(c) What additional restrictions on the parameters would need to be introduced to ensure that $p^* > 0$? Does this restriction also ensure that r^* will be positive? Explain.

(d) Why might $r* < 0$ be an economically rational strategy? (You might be interested to know that negative values of r* in the 1950s resulted in U.S. laws to prohibit such practices.)