Problem 1

Consider the monetary sector of a simple macro model. The market rate of interest (r) is determined in this model by the interaction of supply and demand for money balances (M). Suppose that the demand for money balances is given by $M = D(r, y)$, where y is the exogenous level of national income and $\frac{\partial D}{\partial r} < 0$ and $\frac{\partial D}{\partial y} > 0$. Also assume that the supply of money balances is independent of r, but is shifted by the changes in Federal Reserve discount rate (d), that is: $M = S(d)$, where $\frac{\partial S}{\partial d} < 0$. The market can therefore be represented by the two-equation system:

$$
\begin{aligned}
F^1(M, r; y, d) = M - D(r, y) &= 0 \\
F^2(M, r; y, d) = M - S(d) &= 0
\end{aligned}
$$

Assume that both F^1 and F^2 have continuous partial derivatives (perhaps zero) with respect to both of the endogenous variables (M,r) and both of the exogenous variables (y,d).

- Construct the Jacobian determinant for this system. Check if the system of equations has a valid solution.
- Make use of Cramer's rule to conduct a general comparative static analysis of the effect of a change in the Federal Reserve's discount rate on the equilibrium interest rate (i.e., find the sign of $\partial r/\partial d$). Supplement your answer with a simple diagram that illustrates this effect.

Problem 2

Nina believes that her provision of labor (L) gives her money to spend, but additional leisure (Z) increases her probability of remaining healthy. This probability (ϕ) is also increased by her consumption of preventive healthcare (X). In particular, $\phi(Z, X)$, where ϕ_Z and ϕ_X are pozitive, ϕ_{ZZ} and ϕ_{XX} are negative, and ϕ_{ZX} is zero. Nina must pay c dollars for each unit of preventive care. Prevention is not certain; if she becomes ill, with a probability of $1 - \phi(Z, X)$, she faces a fixed treatment cost of F dollars. Nina wants to find the mix of labor, leisure and preventive health care services (L^*, Z^*, X^*) that will maximize the expected income available for non-medical purchases, given her total time endowment (T) and her current wage (w).

- What is the Lagrangian form of this (equality) constrained maximization problem?
- Find the 4 first-order conditions that must be satisfied by ((L^*, Z^*, X^*, λ^*). What additional information might allow you to solve these first-order conditions for the reduced forms?
- Are the second-order conditions for a maximum satisfied?
- Construct the Jacobian matrix, J, for this system of first-order conditions and find the sign of its determinant.
- Use general comparative static methods to determine the effect of an increase in the parameter c on Nina's consumption of preventive health care services.

Problem 3

The Connecticut Department of Transportation (DOT) plans to construct a highway between two given points. It must decide on the number of lanes to built (n) and the width (w) of each lane. The flow (f) of traffic that each lane can support depends on the lane width, or: $f(w)$, where $f_w > 0$ and $f_{ww} < 0$. The total flow capacity of the road is the number of lanes times the flow per lane, or $n \cdot f(w)$. In addition to a fixed cost of initiating any road construction project, costs increase linearly with the total width ($n \cdot w$)of the roads. Thus, total costs of the project are given by: $anw + c$, where $a > 0$ is the cost per unit of road width and $c > 0$ is the fixed cost. The DOT has a fixed budget (B) for this project and wants total costs to equal this budgeted amount.

- Suppose DOT wants to select n and w so as to maximize total flow capacity, $n \cdot f(w)$, subject to the equality constraint that $B = anw + c$. Ignoring the fact that, in reality, n would have to be an integer, give the constrained maximization problem in Lagrangian form.

- Derive the first-order conditions for a maximum. What do these conditions imply about the value of the expression $(f_w - \lambda a)$ at the optimum? What will be the economic interpretation of λ^* in this particular problem?

- Construct Jacobian matrix and find its determinant. [Hint: you may want to use the information about $(f_w - \lambda a)$ from part (b).] Given the assumed restrictions on parameters and the properties of the function $f(w)$, what is the sign of this determinant?

- Use Cramer's rule to find the general comparative static derivatives: $\partial \lambda^* / \partial a$, $\partial n^* / \partial a$, and $\partial w^* / \partial a$. Be sure to indicate the signs of these expressions. Can you give an intuitive explanation of your result for $\partial w^* / \partial a$?