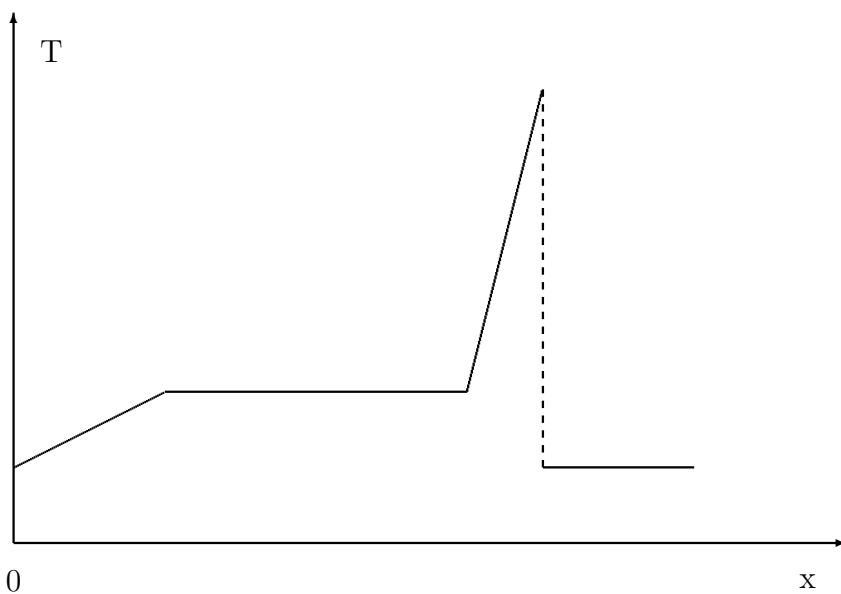


Answer the questions fully to your best ability. Use the space provided. If you run out of room, use the backsides. No partial credit will be given if you do not show the steps of your calculations! **Write as neatly as possible!**

Name: \_\_\_\_\_

1. The following graph shows how your grade for this course depends on the time you devote to study. Thus we can treat this graph as “Total usefulness of studying” (T). Use the blank diagram to plot the graphs that would show:

- (5) (a) The marginal benefit of studying given number of hours.
- (5) (b) The average benefit of studying a given number of hours.



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(2) 2. Evaluate the following expression:  $\lim_{x \rightarrow 0} \frac{x^3 - 2x - c}{x^4} =$

3. Are these functions continuous?

(2) (a)  $y(x) = \begin{cases} 2 - 2x & \text{for } x \leq 5 \\ 2x - 4 & \text{for } x > 5 \end{cases}$

(2) (b)  $y(x) = \frac{1}{x}$

4. Simplify the following expressions:

(2) (a)  $\frac{x+1}{x+2} \cdot \frac{2x^2+7x+3}{x^2+2x+1} \div \frac{2x+1}{x+1}$

Question 4 continues on the next page...

(2) (b)  $((x^{1/3})^8)^{1/2} \cdot x^2 \div x^{3/4}$

(2) 5. Find the roots of the function  $y = x^2 - 1$

6. Consider the matrices:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 3 & 0 \end{pmatrix}, B = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(2) (a) Is  $A + I$  defined? If so, calculate  $A + I$ . If undefined, explain why.

(2) (b) Is  $A \cdot B$  defined? If so, calculate  $A \cdot B$ . If undefined, explain why.

(2) (c) Is  $B \cdot A$  defined? If so, calculate  $B \cdot A$ . If undefined, explain why.

(2) (d) Is  $A \cdot I$  defined? If so, calculate  $A \cdot I$ . If undefined, explain why.

Question 6 continues on the next page...

- (2) (e) Is  $A \cdot A'$  defined? If so, calculate  $A \cdot A'$ . If undefined, explain why.
- (2) (f) Is the determinant of  $I$  defined? If so, calculate the determinant of  $I$ . If undefined, explain why.
- (2) (g) Is  $A'B - B$  defined? If so, calculate  $A'B - B$ . If undefined, explain why.
- (2) (h) Is  $(A'B - B)^{-1}$  defined? If so, calculate  $(A'B - B)^{-1}$ . If undefined, explain why.
- (2) (i) Assume  $\lambda$  is a scalar. Is the determinant of  $\lambda I$  defined? If so, calculate determinant of  $\lambda I$ . If undefined, explain why.
- (2) 7. What are the dimensions of matrix  $M$ , if  $M = (X' \cdot Y)^{-1}$ , and  $X$  is  $5 \times 2$  matrix and  $Y$  is  $5 \times 2$  matrix? Maybe  $(X' \cdot Y)^{-1}$  is undefined?

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8. Consider the matrix  $H = \begin{pmatrix} 4 & 1 & 0 \\ -1 & 0 & 1 \\ 2 & 3 & 0 \end{pmatrix}$

- (2) (a) Find the determinant of  $H$ .
- (2) (b) Find the cofactor matrix  $C$  for matrix  $H$ .
- (2) (c) Find the adjoint matrix,  $adj(H)$  for matrix  $H$ .
- (2) (d) Find  $H^{-1}$ , the inverse of  $H$ .
- (1) (e) Verify that the solution to above is correct.

9. Consider the export/import model:

$$X = 1000 - 20E + 0.2Y_F$$

$$M = 450 + 10E + 0.15Y_D$$

$$X = M$$

where  $X$  represents exports,  $M$  represents imports,  $E$  represents the exchange rate,  $Y_F$  represents foreign income, and  $Y_D$  represents domestic income. The exogenous variables include  $Y_D$  and  $Y_F$ , and endogenous variables are  $X$ ,  $M$ , and  $E$ .

- (4) (a) Rewrite this system of equations in matrix form  $Ax = d$ . [Note: You can eliminate  $X$  or  $M$  from the original system of equations, reducing it to **two equations**.]

- (4) (b) Find the determinant of  $A$ .

- (6) (c) Find the solution to this system of equations. [Hint: You can do it by substitution, or using matrix algebra  $x = A^{-1}d$ .]

