Answer the questions fully to your best ability. Use the space provided. If you run out of room, use the backsides. No partial credit will be given if you do not show the steps of your calculations! **Write as neatly as possible!**

Name: _____

1. It's a sunny summer. You heard that if you stay too long in the sun you may develop a skin cancer, but other doctors say that staying in the sun is good for your skin, since sun induced vitamin D production. The longer you stay in the sun, the higher the probability of skin cancer. Overall the benefit of the time spent in the sun is measured by a function B(t):

$$B(t) = -AW \cdot p(t)t + ct$$

Where t is the time spent in the sun, A is your age, W is the damage caused by skin cancer ($W > 0$), c measures the benefit of vitamin D produced, and $p(t)$ is the probability of developing skin cancer as a function of time spent in the sun ($p_t > 0$, and $p_{tt} > 0$). You would like to figure out how long you should stay in the sun.

(3) (a) Find the first order condition for maximization of $B(t)$.

(2) (b) Verify that the second order condition for a maximum is met.

(1) (c) Can we find the optimal amount of time (t^*) from the first order condition? Explain.

(2) (d) Should you stay in the sun longer as you get older? [Hint: Apply the implicit function theorem to find $\frac{\partial t^*}{\partial A}$]

Question 1 continues on the next page...

(2) (e) How you behavior is going to change if your perceived damage of the skin cancer is going to increase? [Hint: Find $\frac{\partial t^*}{\partial W}$]

2. A monopolist gas producing company wants to find the optimal amount of output (Q) that it needs to produce to maximize its profit. The demand for oil is a function of the income (y) of an average consumer, the price (p) that the producer gets per gallon of gas sold and the sales tax (t). The demand function for gas is: $Q = a - bp \cdot (1+t) + d\ln(y)$. The producer experiences a fixed cost of production F and a marginal cost per unit produced c. So that the profit function that needs to be maximized is:

$$\Pi(Q) = p(Q) \cdot Q - c \cdot Q - F$$

In order to find the explicit profit function in terms of output we need to find the inverse demand function $p(Q)$, which is $p(Q) = \frac{a}{b\cdot(t+1)} + \frac{d\ln(y)}{b\cdot(t+1)} - \frac{1}{b\cdot(t+1)}Q$. So that the profit function is:

$$\Pi(Q) = \left(\frac{a}{b\cdot(t+1)} + \frac{d\ln(y)}{b\cdot(t+1)} - \frac{1}{b\cdot(t+1)}Q\right)\cdot Q - c\cdot Q - F$$

All the parameters of the problem are positive: $a, b, c, d, t > 0$

(3) (a) Find the first order condition for profit maximization.

(2) (b) Solve for optimal amount of gas Q^* that maximizes profit of gas producer.

Question 2 continues on the next page...

(2) (c) Verify that the second order condition for profit maximization is met.

(1) (d) Find the comparative static derivatives Q_t^* and Q_y^*. What are the signs of these derivatives? Provide an economic interpretation of your results. [Hint: You can apply the implicit function theorem, but there is no need for it.]

(2) (e) Find $\epsilon_{Q,p}$ the elasticity of demand for gas with respect to a change in price. **Evaluate $\epsilon_{Q,p}$ at Q^*.** [Hint: You can check your answer to see that $\epsilon_{Q,p} = -1$ in case marginal cost is zero.]

Plotting cost functions:

3. Total cost function $TC = 3x^3 - 20x^2 + 50x$

(2) (a) Find the stationary points of $TC = 3x^3 - 20x^2 + 50x$. Verify each of them, if it is a maximum, a minimum or an inflection point.

(2) (b) Find a range of x in which TC is increasing and a range of x in which TC is decreasing. [Hint: $TC_x > 0$ if TC is increasing and $TC_x < 0$ if TC is decreasing.]

(2) (c) Find a range of x in which TC is concave, and a range of x in which TC is convex [Hint: $TC_{xx} < 0$ if TC is concave and $TC_{xx} > 0$ if TC is convex.]

Question 3 continues on the next page...

(2) (d) Use the information you obtained about TC to plot a sketch of this function.

4. Average cost function $AC = TC/Q = 3x^2 - 20x + 50$

(2) (a) Find the stationary points of $AC = 3x^2 - 20x + 50$. Verify each of them, if it is a maximum, a minimum or an inflection point.

(2) (b) Find a range of x in which AC is increasing and a range of x in which AC is decreasing. [Hint: $AC_x > 0$ if AC is increasing and $AC_x < 0$ if AC is decreasing.]

Question 4 continues on the next page...

(2) (c) Find a range of x in which AC is concave, and a range of x in which AC is convex [Hint: $AC_{xx} < 0$ if AC is concave and $AC_{xx} > 0$ if AC is convex.]

(2) (d) Use the information you obtained about AC to plot a sketch of this function.

5. Marginal cost function $MC = \frac{dTC}{dQ} = 9x^2 - 40x + 50$

(2) (a) Find the stationary points of $MC = 9x^2 - 40x + 50$. Verify each of them, if it is a maximum, a minimum or an inflection point.

Question 5 continues on the next page...

(2) (b) Find a range of x in which MC is increasing and a range of x in which MC is decreasing. [Hint: $MC_x > 0$ if MC is increasing and $MC_x < 0$ if MC is decreasing.]

(2) (c) Find a range of x in which MC is concave, and a range of x in which MC is convex [Hint: $MC_{xx} < 0$ if MC is concave and $MC_{xx} > 0$ if MC is convex.

(2) (d) Use the information you obtained about MC to plot a sketch of this function.

(1) 6. Plot TC, AC and MC on one graph.

7. Beavis is a very simple person. The only things in life he cares about are beer and music CDs. His happiness is measured by a specific "happiness function" $W(x, y) = a\ln(x) + b\ln(y)$, where x is amount of beer consumed and y is amount of CDs bought. The price of one bottle of beer is d and the price of 1 CD is c. Beavis does not work anywhere, but he has a good roommate Butthead, who gives him m dollars each month. Thus whatever amount of beer and CDs Beavis decides to buy, his expenditure must not exceed m: $dx + cy \leq m$. How should Beavis spend the money he gets in order to maximize his happiness?

(2) (a) Write the Lagrangian for this problem. Note, that Beavis has no interest in money itself and he will spend everything. Thus $dx + cy = m$.

(2) (b) Find the first order conditions for happiness maximization.

(2) (c) Solve the system of FOCs for the optimal consumption of beer and CDs.

Question 7 continues on the next page...

(2) (d) Verify that the solution actually maximizes happiness instead of minimizing it.

(2) (e) How would Beavis change his allocation of money if the price of CDs increases? Will he increase his consumption of beer? What about CDs?

8. Butthead is a complicated personality. Like Beavis, he also cares about beer and CDs only, but he has no clue how much does he like each of these things (Oh well... he is a butt-head!). All he knows is that more of CD's is better than less and that more of beer is better than less. On top of that he knows *some* math. (Knowing something, sometimes is worse than not knowing it at all). So he thinks, that his happiness is *some* function $W(x, y)$, where x is the amount of beer consumed and y is the amount of CDs bought. Remember, he has no idea what exactly is the functional form for $W(x, y)$, but he knows that this function has the following properties: $W_x > 0$, $W_y > 0$, $W_{xx} < 0$, $W_{yy} < 0$ and $W_{xy} = 0$. Butthead is still at school and his only income is the money (T) he collects (most probably forcefully) from his classmates. He also gives a little bit (m) to Beavis, so that his total income is $T - m$. How should Butthead spend his income if the price of one bottle of beer is b and the price of 1 CD is c?

(2) (a) Write the Lagrangian for this problem. (Like Beavis, Butthead has no interest in money itself, thus he will spend all of his income: $bx + cy = T - m$.)

(2) (b) Find the first order conditions for happiness maximization.

Question 8 continues on the next page...

(1) (c) Can we solve for optimal x^* and y^* explicitly? Explain.

(2) (d) Verify that the system of the FOCs for the optimal consumption of beer and CDs will lead to x^* and y^* that is going to maximize happiness instead of minimizing it.

(3) (e) Can we at least say if Butthead is going to increase his consumption of beer if he decided to give more money to Beavis? [Hint: find $\frac{\partial x^*}{\partial m}$. This involves finding the Jacobian, and using the Cramer's rule.]

9. Please tell me what you *did not like* about this class.

(0) (a) What subject was the hardest to comprehend?

(0) (b) What you did not like about my teaching style?

(0) (c) What should I have stressed more?

(0) (d) What is your expected grade? [Be honest!]

Grade table

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	10	10	8	8	8	1	10	10	0	65
Score:										