

5. a. The demand for labor is determined by the amount of labor that a profit-maximizing firm wants to hire at a given real wage. The profit-maximizing condition is that the firm hire labor until the marginal product of labor equals the real wage,

$$MPL = \frac{W}{P}.$$

The marginal product of labor is found by differentiating the production function with respect to labor (see the appendix to Chapter 3 for more discussion),

$$\begin{aligned} MPL &= \frac{dY}{dL} \\ &= \frac{d(K^{1/3}L^{2/3})}{dL} \\ &= \frac{2}{3} K^{1/3}L^{-1/3}. \end{aligned}$$

In order to solve for labor demand, we set the MPL equal to the real wage and solve for L :

$$\begin{aligned} \frac{2}{3} K^{1/3}L^{-1/3} &= \frac{W}{P} \\ L &= \frac{8}{27} K \left(\frac{W}{P}\right)^{-3}. \end{aligned}$$

Notice that this expression has the intuitively desirable feature that increases in the real wage reduce the demand for labor.

- b. We assume that the 1,000 units of capital and the 1,000 units of labor are supplied inelastically (i.e., they will work at any price). In this case we know that all 1,000 units of each will be used in equilibrium, so we can substitute them into the above labor demand function and solve for $\frac{W}{P}$.

$$\begin{aligned} 1,000 &= \frac{8}{27} 1,000 \left(\frac{W}{P}\right)^{-3} \\ \frac{W}{P} &= \frac{2}{3}. \end{aligned}$$

In equilibrium, employment will be 1,000, and multiplying this by $2/3$ we find that the workers earn 667 units of output. The total output is given by the production function:

$$\begin{aligned} Y &= K^{1/3}L^{2/3} \\ &= 1,000^{1/3}1,000^{2/3} \\ &= 1,000. \end{aligned}$$

Notice that workers get two-thirds of output, which is consistent with what we know about the Cobb–Douglas production function from the appendix to Chapter 3.

- c. The congressionally mandated wage of 1 unit of output is above the equilibrium wage of $2/3$ units of output.
 d. Firms will use their labor demand function to decide how many workers to hire at the given real wage of 1 and capital stock of 1,000:

$$\begin{aligned} L &= \frac{8}{27} 1,000(1)^{-3} \\ &= 296, \end{aligned}$$

so 296 workers will be hired for a total compensation of 296 units of output.

- e. The policy redistributes output from the 704 workers who become involuntarily unemployed to the 296 workers who get paid more than before. The lucky workers benefit less than the losers lose as the total compensation to the working class falls from 667 to 296 units of output.
- f. This problem does focus the analysis of minimum-wage laws on the two effects of these laws: they raise the wage for some workers while downward-sloping labor demand reduces the total number of jobs. Note, however, that if labor demand is less elastic than in this example, then the loss of employment may be smaller, and the change in worker income might be positive.