Chapter 7 learning objectives

- Learn the closed economy Solow model
- See how a country's standard of living depends on its saving and population growth rates
- Learn how to use the "Golden Rule" to find the optimal savings rate and capital stock

How Solow model is different from Chapter 3's model

1. ***K*** is no longer fixed:
   investment causes it to grow,
   depreciation causes it to shrink.
2. ***L*** is no longer fixed:
   population growth causes it to grow.
3. The consumption function is simpler.

How Solow model is different from Chapter 3's model

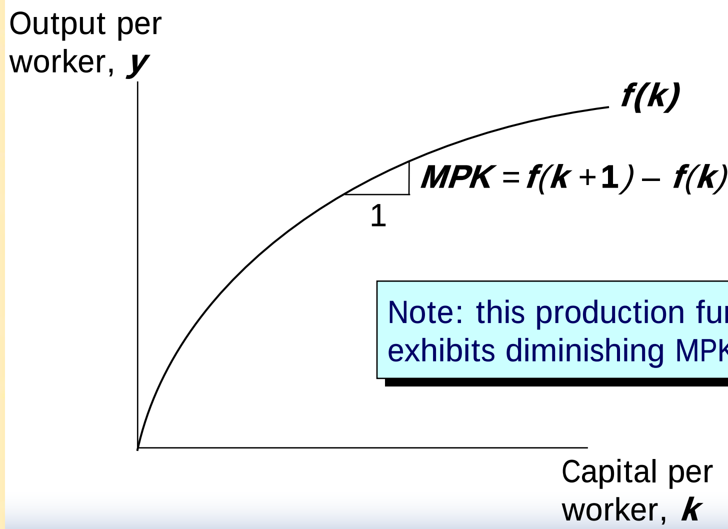
4. No G or T
   (only to simplify presentation;
   we can still do fiscal policy experiments)

5. Cosmetic differences.

The production function

- In aggregate terms: $Y = F(K, L)$
- Define: $y = Y/L$ = output per worker
  $k = K/L$ = capital per worker
- Assume constant returns to scale:
  $zY = F(zK, zL)$ for any $z > 0$
- Pick $z = 1/L$. Then
  $Y/L = F(K/L, 1)$
  $y = F(k, 1)$
  $y = f(k)$ where $f(k) = F(k, 1)$

The production function



Note: this production function exhibits diminishing MPK.

The national income identity

- $Y = C + I$ (*remember, no* G)
- In "per worker" terms:

$$y = c + i$$

where $c = C/L$ and $i = I/L$

The consumption function

- s = the saving rate,
  the fraction of income that is saved
  (s is an exogenous parameter)

  Note: s is the only lowercase variable
  that is not equal to
  its uppercase version divided by L

- Consumption function: $c = (1-s)y$
  (per worker)

Saving and investment

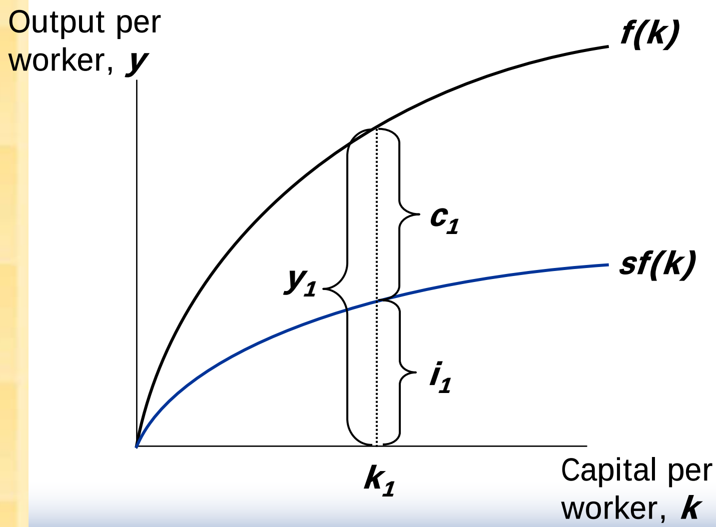
- saving (per worker) $= y - c$
  $= y - (1-s)y$
  $= sy$
- National income identity is $y = c + i$

  Rearrange to get: $i = y - c = sy$

  (investment = saving, like in chap. 3!)

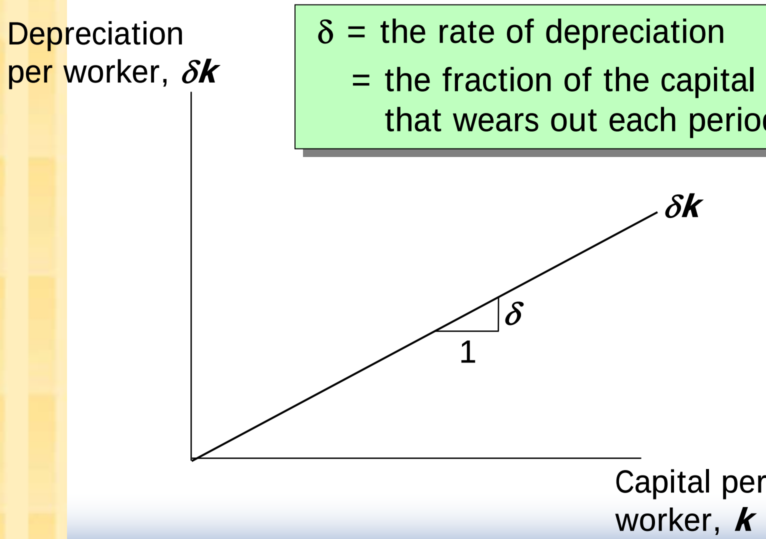
- Using the results above,
  $i = sy = sf(k)$

Output, consumption, and investment



Depreciation

δ = the rate of depreciation
= the fraction of the capital stock that wears out each period



Capital accumulation

The basic idea:

Investment makes the capital stock bigger, depreciation makes it smaller.

Capital accumulation

Change in capital stock = investment – depreciation

$\Delta k = i - \delta k$

Since $i = sf(k)$, this becomes:

$$\Delta k = s\,f(k) - \delta k$$

The equation of motion for k

$$\Delta k = s f(k) - \delta k$$

- the Solow model's central equation
- Determines behavior of capital over time…
- …which, in turn, determines behavior of all of the other endogenous variables because they all depend on k. E.g.,

  income per person: $y = f(k)$

  consump. per person: $c = (1-s) f(k)$

The steady state

$$\Delta k = s f(k) - \delta k$$

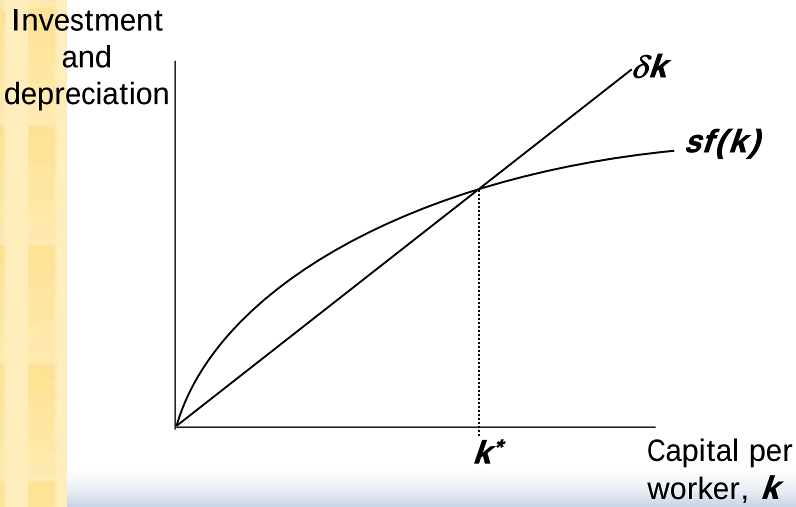
If investment is just enough to cover depreciation [$sf(k) = \delta k$],

then capital per worker will remain constant:

$$\Delta k = 0.$$

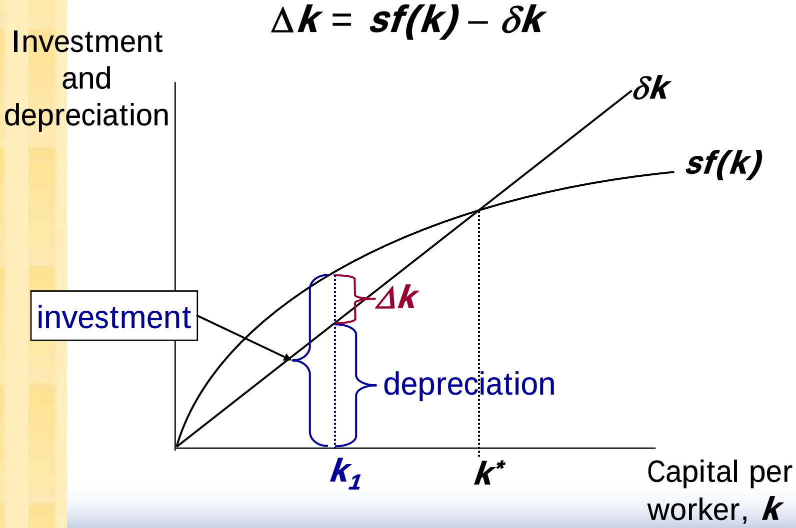
This constant value, denoted k^*, is called the ***steady state capital stock***.

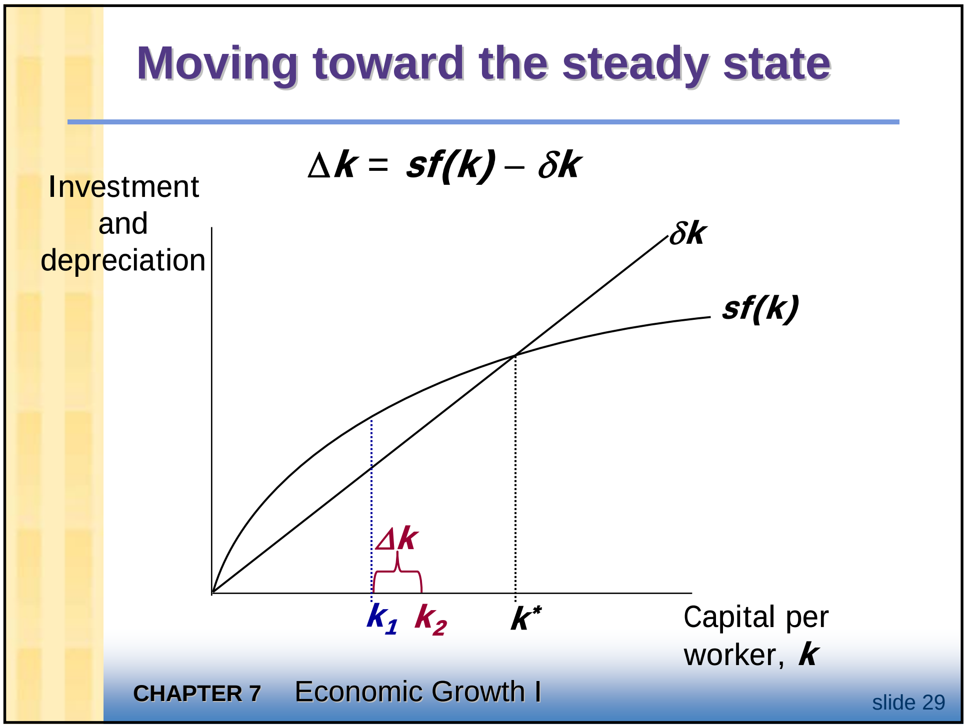
The steady state



Moving toward the steady state

$$\Delta k = sf(k) - \delta k$$




Moving toward the steady state
Δk = sf(k) – δk
Investment and depreciation
δk
sf(k)
Δk
k1
k2
k*
Capital per worker, k
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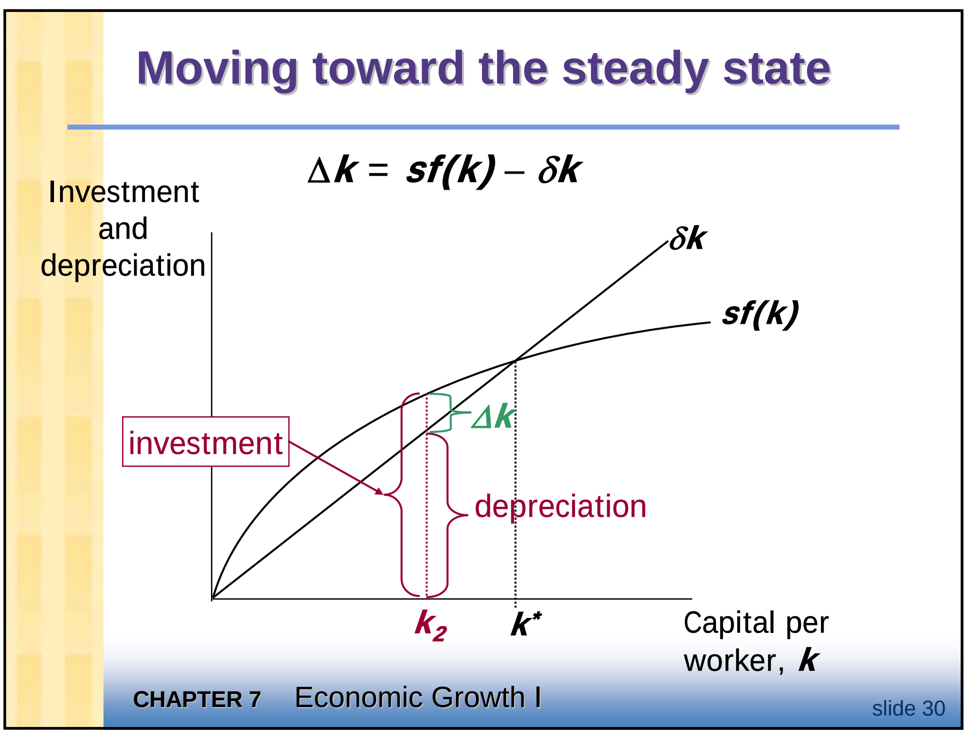
Moving toward the steady state
Δk = sf(k) – δk
Investment and depreciation
δk
sf(k)
investment
Δk
depreciation
k2
k*
Capital per worker, k
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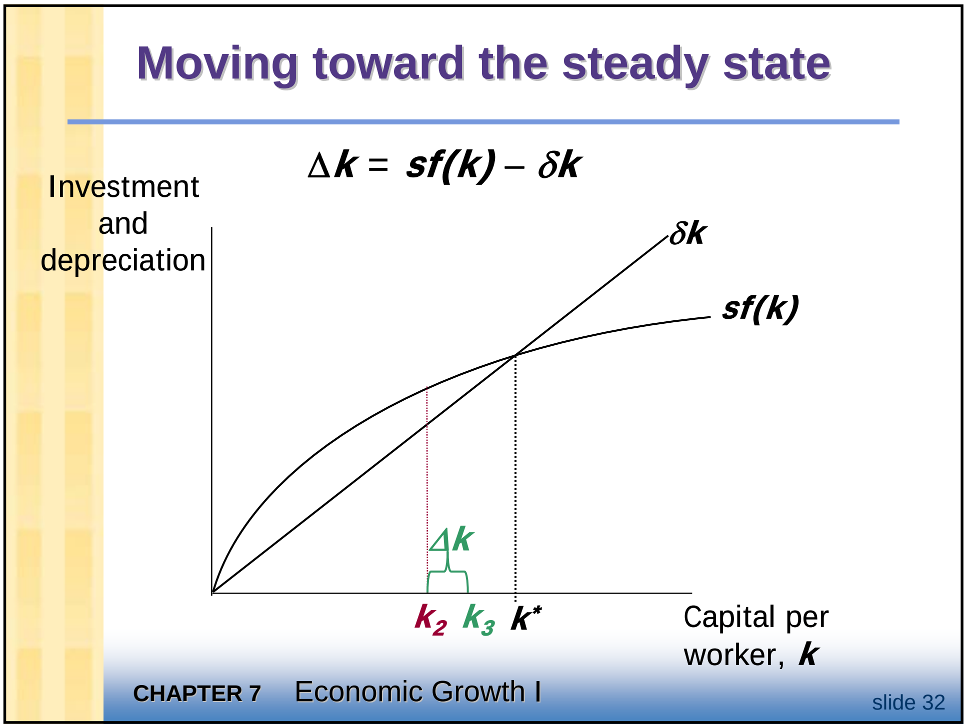
Moving toward the steady state
Δk = sf(k) – δk
Investment and depreciation
δk
sf(k)
Δk
k2 k3 k*
Capital per worker, k
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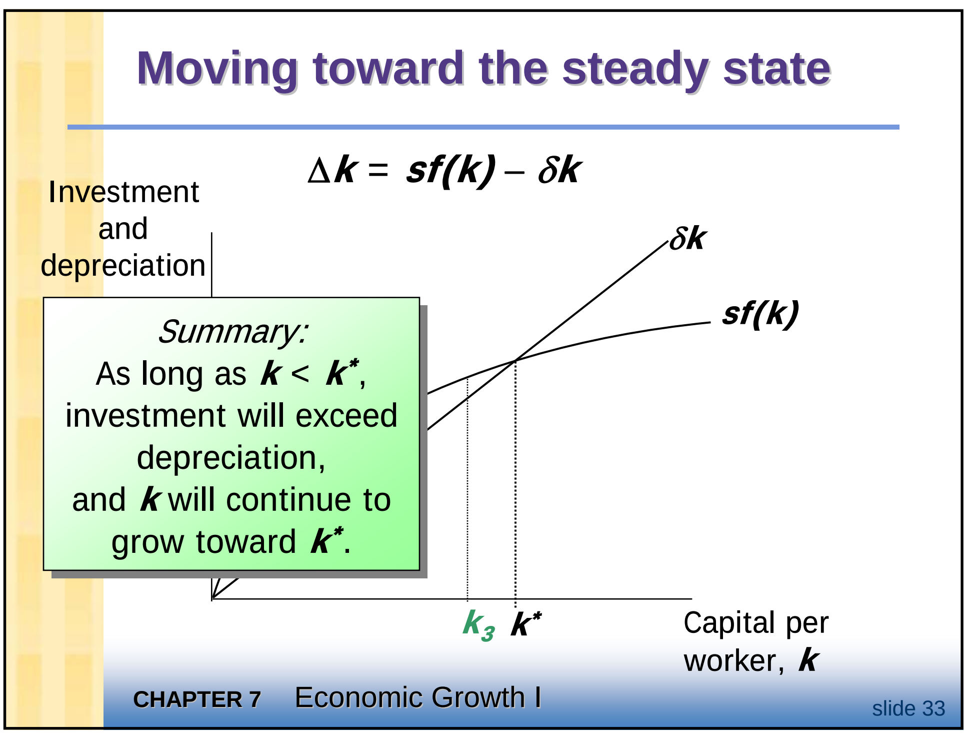
Moving toward the steady state
Δk = sf(k) – δk
Investment and depreciation
δk
sf(k)
Summary:
As long as k < k*,
investment will exceed depreciation,
and k will continue to grow toward k*.
k3 k*
Capital per worker, k
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Now you try:

Draw the Solow model diagram, labeling the steady state k^*.

On the horizontal axis, pick a value greater than k^* for the economy's initial capital stock. Label it k_1.

Show what happens to k over time. Does k move toward the steady state or away from it?

A numerical example

Production function (aggregate):

$$Y = F(K, L) = \sqrt{K \times L} = K^{1/2}L^{1/2}$$

To derive the per-worker production function, divide through by L:

$$\frac{Y}{L} = \frac{K^{1/2}L^{1/2}}{L} = \left(\frac{K}{L}\right)^{1/2}$$

Then substitute $y = Y/L$ and $k = K/L$ to get

$$y = f(k) = k^{1/2}$$

A numerical example, *cont.*

Assume:

- ***s*** = 0.3
- δ = 0.1
- initial value of ***k*** = 4.0

Approaching the Steady State: A Numerical Example

Assumptions: $y = \sqrt{k}$; $s = 0.3$; $\delta = 0.1$; initial $k = 4.0$

Year	***k***	***y***	***c***	***i***	***δk***	***Δk***
1	4.000	2.000	1.400	0.600	0.400	0.200
2	4.200	2.049	1.435	0.615	0.420	0.195
3	4.395	2.096	1.467	0.629	0.440	0.189

Approaching the Steady State: A Numerical Example

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3	4.395	2.096	1.467	0.629	0.440	0.189
4	4.584	2.141	1.499	0.642	0.458	0.184
…						
10	5.602	2.367	1.657	0.710	0.560	0.150
…						
25	7.351	2.706	1.894	0.812	0.732	0.080
…						
100	8.962	2.994	2.096	0.898	0.896	0.002
…						
∞	9.000	3.000	2.100	0.900	0.900	0.000

Exercise: solve for the steady state

Continue to assume

$s = 0.3$, $\delta = 0.1$, and $y = k^{1/2}$

Use the equation of motion

$$\Delta k = s f(k) - \delta k$$

to solve for the steady-state values of ***k***, ***y***, and ***c***.

Solution to exercise:

$\Delta k = 0$	def. of steady state
$s\,f(k^*) = \delta k^*$	eq'n of motion with $\Delta k = 0$
$0.3\sqrt{k^*} = 0.1k^*$	using assumed values

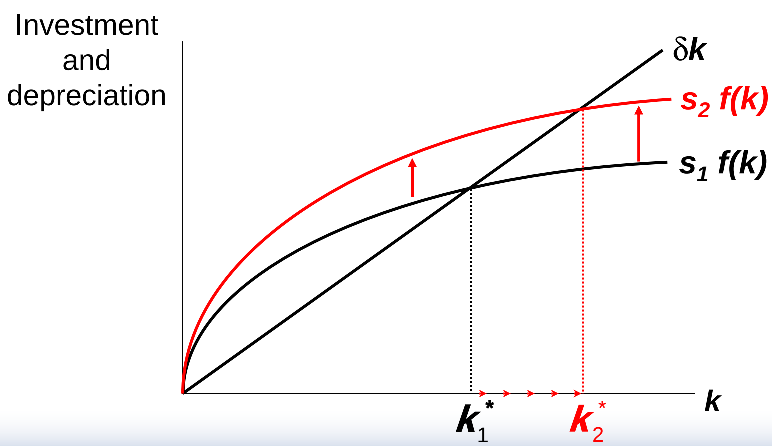
$$3 = \frac{k^*}{\sqrt{k^*}} = \sqrt{k^*}$$

Solve to get: $k^* = 9$ and $y^* = \sqrt{k^*} = 3$

Finally, $c^* = (1-s)y^* = 0.7 \times 3 = 2.1$

An increase in the saving rate

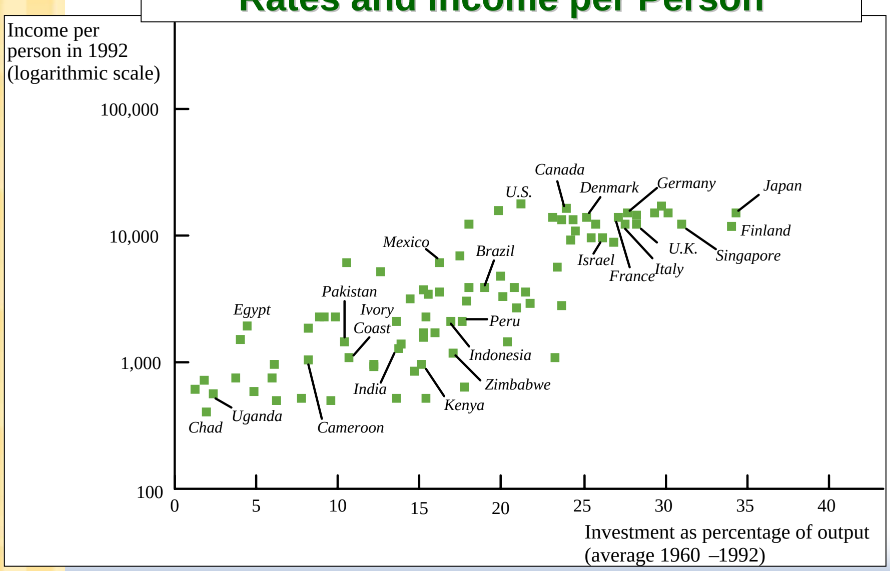
An increase in the saving rate raises investment…
…causing the capital stock to grow toward a new steady state:



Prediction:

- Higher s ⇒ higher k^*.
- And since $y = f(k)$,
  higher k^* ⇒ higher y^* .
- Thus, the Solow model predicts that countries with higher rates of saving and investment will have higher levels of capital and income per worker in the long run.

International Evidence on Investment Rates and Income per Person



The Golden Rule: introduction

- Different values of s lead to different steady states. How do we know which is the "best" steady state?
- Economic well-being depends on consumption, so the "best" steady state has the highest possible value of consumption per person: $c^* = (1-s)\, f(k^*)$
- An increase in s
  - leads to higher k^* and y^*, which may raise c^*
  - reduces consumption's share of income $(1-s)$, which may lower c^*
- So, how do we find the s and k^* that maximize c^*?

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The Golden Rule Capital Stock

k^*_{gold} = the **Golden Rule level of capital,** the steady state value of k that maximizes consumption.

To find it, first express c^* in terms of k^*:

$$c^* = y^* - i^*$$
$$= f(k^*) - i^*$$
$$= f(k^*) - \delta k^*$$

In general:
$i = \Delta k + \delta k$

In the steady state:
$i^* = \delta k^*$
because $\Delta k = 0$.

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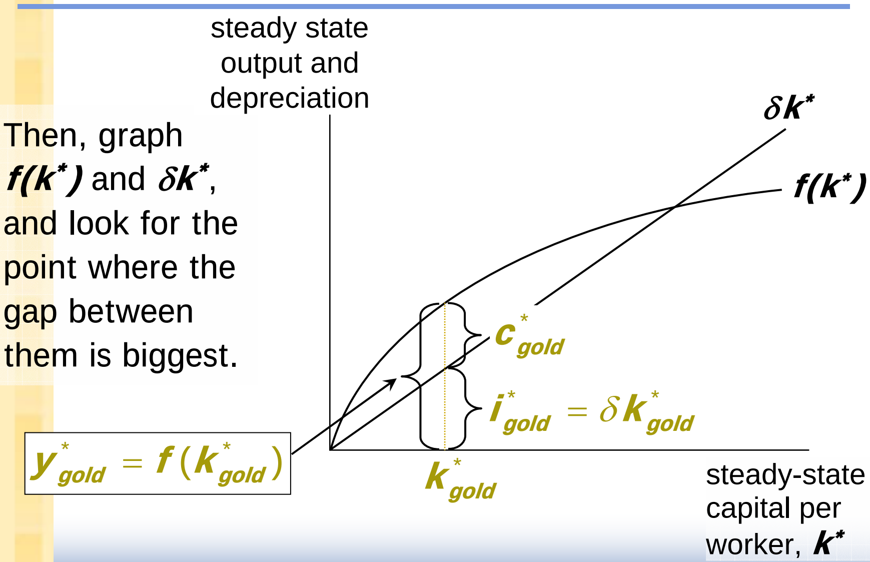
The Golden Rule Capital Stock

Then, graph $f(k^*)$ and δk^*, and look for the point where the gap between them is biggest.



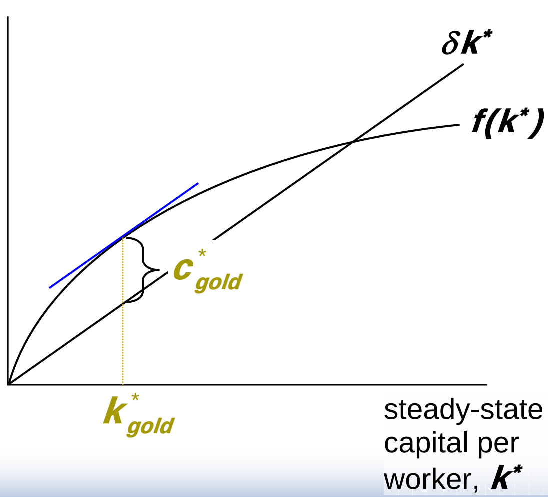
The Golden Rule Capital Stock

$c^* = f(k^*) - \delta k^*$ is biggest where the slope of the production func.

equals

the slope of the depreciation line:

MPK = δ



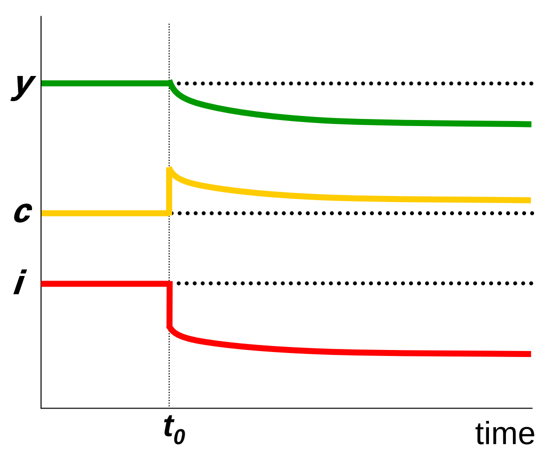
The transition to the Golden Rule Steady State

- The economy does NOT have a tendency to move toward the Golden Rule steady state.
- Achieving the Golden Rule requires that policymakers adjust ***s***.
- This adjustment leads to a new steady state with higher consumption.
- But what happens to consumption during the transition to the Golden Rule?

Starting with too much capital

If $k^* > k^*_{gold}$ then increasing c^* requires a fall in ***s***.

In the transition to the Golden Rule, consumption is higher at all points in time.

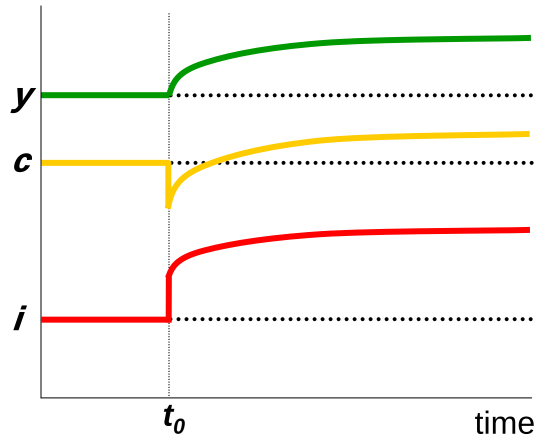


Starting with too little capital

If $k^* < k^*_{gold}$

then increasing c^* requires an increase in s.

Future generations enjoy higher consumption, but the current one experiences an initial drop in consumption.



Population Growth

- Assume that the population--and labor force--grow at rate n. (n is exogenous)

$$\frac{\Delta L}{L} = n$$

- EX: Suppose L = 1000 in year 1 and the population is growing at 2%/year (n = 0.02).

  Then $\Delta L = n\,L = 0.02 \times 1000 = 20$,
  so L = 1020 in year 2.

Break-even investment

$(\delta + n)\boldsymbol{k}$ = **break-even investment**, the amount of investment necessary to keep $\boldsymbol{k}$ constant.

Break-even investment includes:

- $\delta\boldsymbol{k}$ to replace capital as it wears out
- $\boldsymbol{nk}$ to equip new workers with capital (*otherwise,* $\boldsymbol{k}$ *would fall as the existing capital stock would be spread more thinly over a larger population of workers*)

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The equation of motion for *k*

- With population growth, the equation of motion for $\boldsymbol{k}$ is

$$\Delta k = \underbrace{s f(k)}_{\text{actual investment}} - \underbrace{(\delta + n)\, k}_{\text{break-even investment}}$$

actual investment

break-even investment

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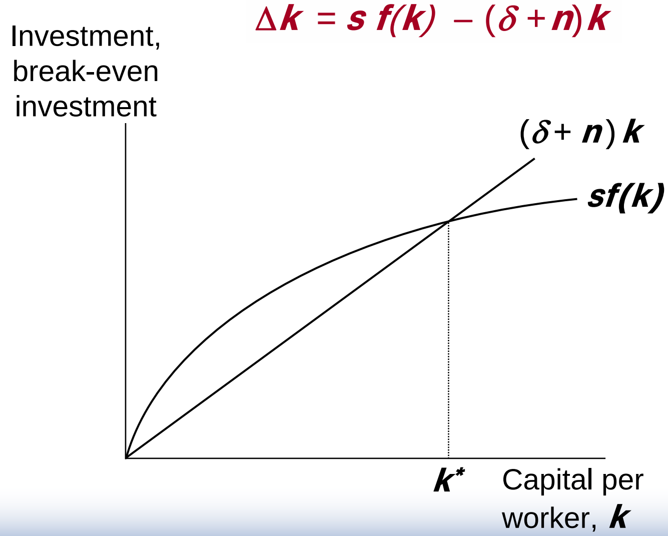
The Solow Model diagram

$\Delta k = s\,f(k) - (\delta + n)k$



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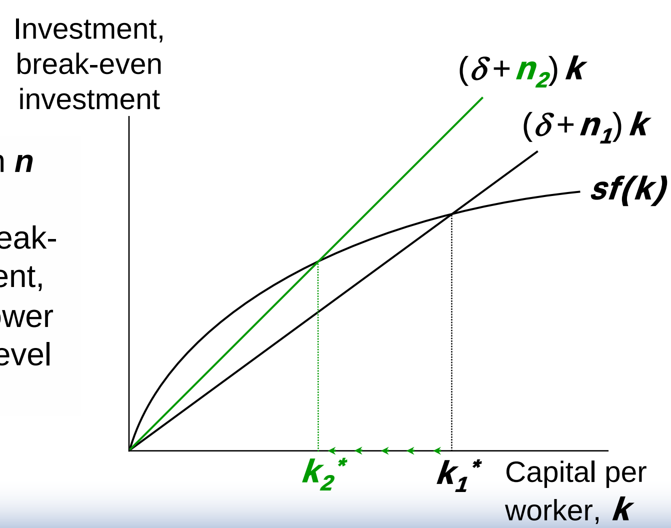
The impact of population growth

An increase in ***n*** causes an increase in break-even investment, leading to a lower steady-state level of ***k***.



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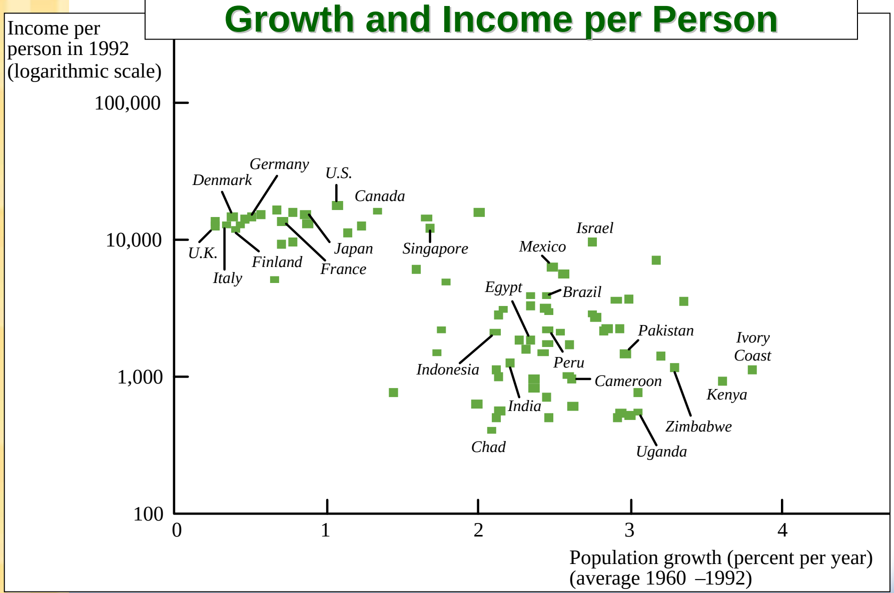
Prediction:

- Higher $\boldsymbol{n}$ $\Rightarrow$ lower $\boldsymbol{k^*}$.
- And since $\boldsymbol{y} = \boldsymbol{f(k)}$, lower $\boldsymbol{k^*}$ $\Rightarrow$ lower $\boldsymbol{y^*}$.
- Thus, the Solow model predicts that countries with higher population growth rates will have lower levels of capital and income per worker in the long run.

International Evidence on Population Growth and Income per Person



The Golden Rule with Population Growth

To find the Golden Rule capital stock, we again express c^* in terms of k^*:

$$c^* = y^* - i^*$$
$$= f(k^*) - (\delta + n)\,k^*$$

c^* is maximized when

$$\text{MPK} = \delta + n$$

or equivalently,

$$\boxed{\text{MPK} - \delta = n}$$

In the Golden Rule Steady State, the marginal product of capital net of depreciation equals the population growth rate.

Chapter Summary

1. The Solow growth model shows that, in the long run, a country's standard of living depends
   - positively on its saving rate.
   - negatively on its population growth rate.
2. An increase in the saving rate leads to
   - higher output in the long run
   - faster growth temporarily
   - but not faster steady state growth.

Chapter Summary

3. If the economy has more capital than the Golden Rule level, then reducing saving will increase consumption at all points in time, making all generations better off.

   If the economy has less capital than the Golden Rule level, then increasing saving will increase consumption for future generations, but reduce consumption for the present generation.